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Reducing the sensitivity of the Mølmer-Sørensen gate for ion-trap quantum computing to unbalanced laser intensities

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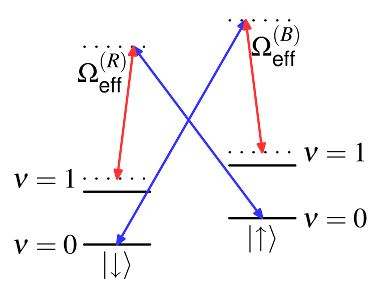
The Mølmer-Sørensen (MS) gate



$H_1 = \hbar\chi\hat{q}_\phi(t)\,\sigma_x,$
$\hat{q}_\phi = e^{i\phi}a + e^{-i\phi}a^\dagger$

The MS, or σ_x, gate uses two pairs of Raman beams of equal strength to apply opposite forces to ions in two orthogonal superpositions of the qubit states [1].

Compared to the σ_z-gate [2] this

- requires at least one additional laser beam,
- works for field-independent qubits.

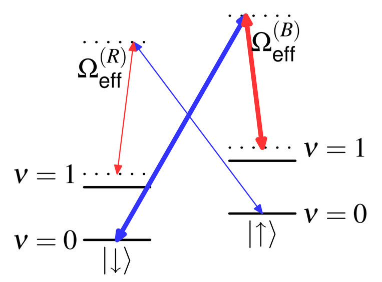
Two qubit gate

The two-ion Hamiltonian for the MS gate is equivalent to H_z, except the roles of $|\uparrow\rangle$ ($|\downarrow\rangle$) and $|x+\rangle$ ($|x-\rangle$) are interchanged:

$$H_{MS} = \hbar\chi\hat{q}_\phi(t)\left(\sigma_x^{(1)} - \sigma_x^{(2)}\right),$$

for certain ion spacings.

Effects of unbalanced beams



$H_1' = \hbar\chi\hat{q}_{\phi(t)}\,\sigma_x$
$\quad + \varepsilon\hbar\chi\hat{q}_{\phi(t)+\pi/2}\,\sigma_y$

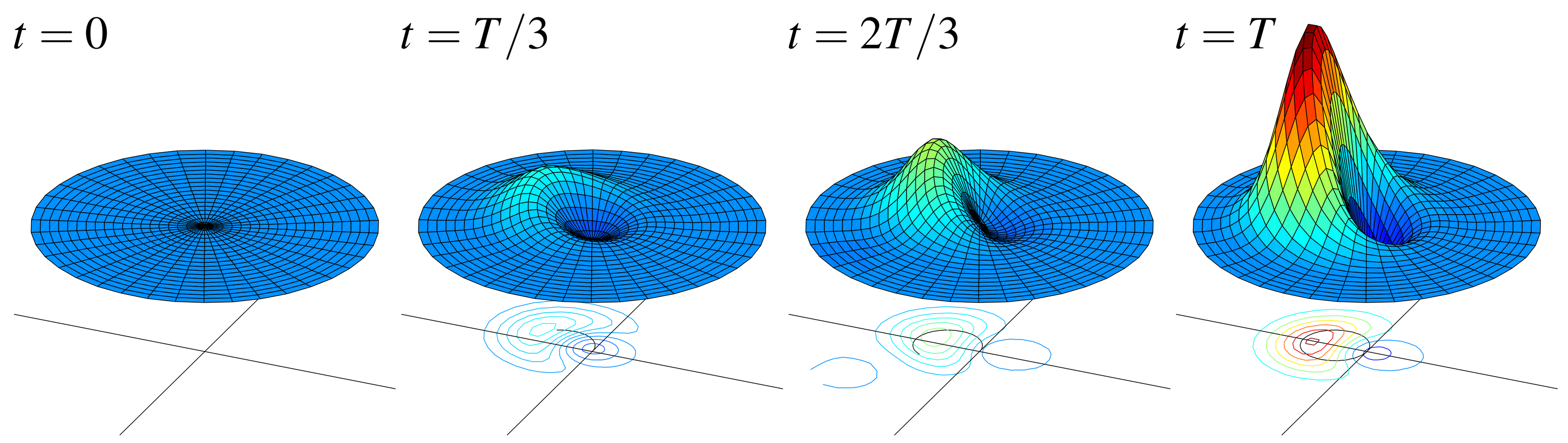
If $\varepsilon \equiv (\Omega_{\text{eff}}^{(R)} - \Omega_{\text{eff}}^{(B)})/(\Omega_{\text{eff}}^{(R)} + \Omega_{\text{eff}}^{(B)}) \neq 0$, H_{MS} is modified to read

$$\begin{aligned} H_{MS}' &= H_{MS} + \varepsilon\hbar\chi\hat{q}_{\phi(t)+\pi/2}\,(\sigma_y^{(1)} - \sigma_y^{(2)}) \\ &= H_{MS} + \varepsilon\, 2i\hbar\chi\hat{q}_{\phi(t)+\pi/2}\left(\left|\Psi^-\right\rangle\left\langle\Phi^+\right| - \left|\Phi^+\right\rangle\left\langle\Psi^-\right|\right) \end{aligned}$$

where $|\Psi^-\rangle \propto |x+\rangle|x-\rangle - |x-\rangle|x+\rangle$ and $|\Phi^+\rangle \propto |x+\rangle|x+\rangle + |x-\rangle|x-\rangle$.

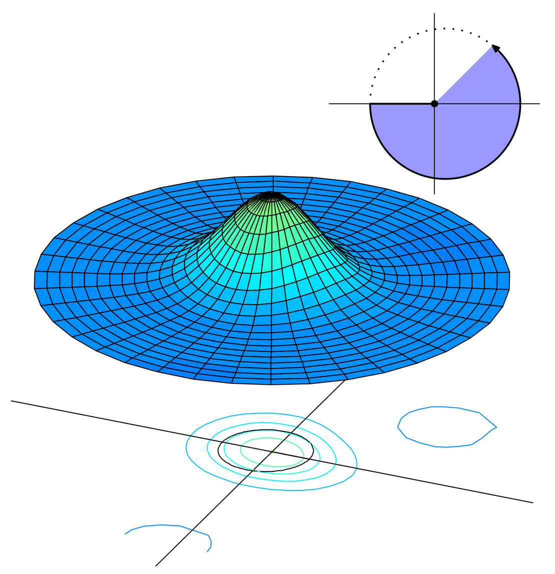
The error term can be described as flipping ions between the interacting and non-interacting states during the phase space traversal:

$$\left\langle\Phi^+\middle|U_{MS}'(t)\left(\middle|x+\right\rangle\middle|x-\right\rangle\Big) = -\varepsilon\sqrt{2}\chi\int_0^t \hat{q}_{\phi(t')+\pi/2}e^{i\Phi(t')}D(\alpha(t'))dt' + \mathcal{O}(\varepsilon^2) = \varepsilon M_{\text{flip}}\{\alpha\}.$$



The motional state of flipped ions: Surfaces, and colored contour lines below, show the Wigner-distribution of $M_{\text{flip}}\{\alpha\}\,|0\rangle$ for the phase space path of the $|x+\rangle|x-\rangle$ state during the MS gate, truncated at 0, $1/3$, $2/3$, and $3/3$ of the total gate duration, T. The traversed part of $\alpha(t)$ is shown in black.

Countering the effects of unbalanced beams by pre-shifting the ions



By adding two laser pulses to the gate operation, we can offset the phase space path, thus centering the motional state of the flipped ions on the motional ground state:

- More complicated gate implementation
- No entanglement of motion and qubit states

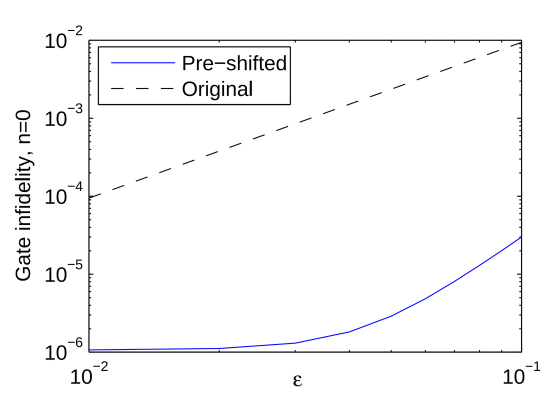
Refocusing

Pre-shifting prevents entanglement of internal and motional states, but $\varepsilon \neq 0$ still causes imperfect evolution of the internal states. This imperfect evolution can be refocused by a Bloch sphere rotation $\mathscr{R}((\hat{x}+\hat{y})/\sqrt{2},\pi)$, interchanging the population in the internal states $|x+\rangle|x-\rangle + |x-\rangle|x+\rangle$ and $|x+\rangle|x+\rangle + |x-\rangle|x-\rangle$, so that the full gate operation is described by

$$U_{RF} = \mathscr{R}((\hat{x}+\hat{y})/\sqrt{2},-\pi)\; U_{MS}'\; \mathscr{R}((\hat{x}+\hat{y})/\sqrt{2},\pi)\; U_{MS}',$$

which for $\Phi(T) = \pi/2$ is a universal gate with the same entangling power as the CNOT-gate.

Outlook



At $\varepsilon = 0.05$, pre-shifting improves the error rate of the MS gate from 10^{-3} to 10^{-5}, below the threshold for scalable quantum computing.

The pre-shifting procedure does not change the phase relations of the gate operation, and the analysis of Ref. [3] still applies.

Acknowledgements

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References

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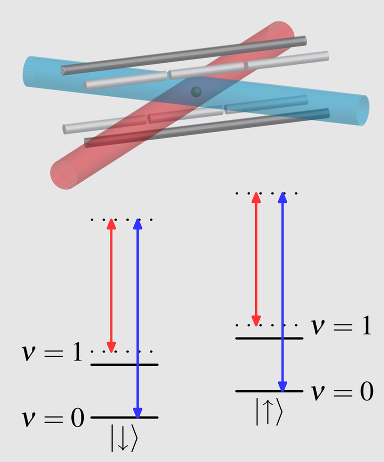
[2] D. Leibfried, B. DeMarco, V. Meyer, D. Lucas, M. Barrett, J. Britton, W. M. Itano, B. Jelenkovic, C. Langer, T. Rosenband, et al., Nature **422**, 412 (2003).
[3] P. J. Lee, K.-A. Brickman, L. Deslauriers, P. C. Haljan, L.-M. Duan, and C. Monroe, *Phase control of trapped ion quantum gates* (2005), quant-ph/0505203.

State dependent forces in the σ_z-basis



$H_{1,z} = \hbar\chi\hat{q}_\phi\,\sigma_z$

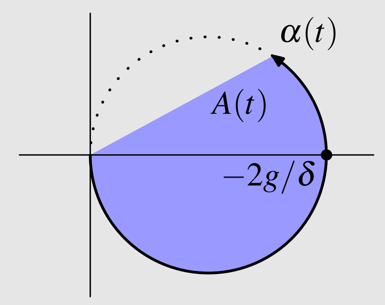
A cold ion interacting with two Raman laser beams will experience a force, $F(t)$, oscillating with the difference frequency of the Raman beams. In the interaction picture with respect to the ion oscillation frequency ω_z, the force, $F(t) = F_0\cos(\omega_z t + \phi(t))$, is described by the Hamiltonian

$$H_{1,z} = \left(-\frac{F_\uparrow z_0}{2}\right)\hat{q}_\phi\,|\uparrow\rangle\langle\uparrow| + \left(-\frac{F_\downarrow z_0}{2}\right)\hat{q}_\phi\,|\downarrow\rangle\langle\downarrow|,$$

where $\hat{q}_\phi = e^{i\phi}a + e^{-i\phi}a^\dagger$ is the ϕ-quadrature of the motional state.

If the qubit is not encoded in a field-insensitive state, beam phases and ion distances can be arranged so that $F_\uparrow = -F_\downarrow$ [2, 3].

Geometric phases in the harmonic oscillator



$H = \hbar g\hat{q}_{\delta t}$

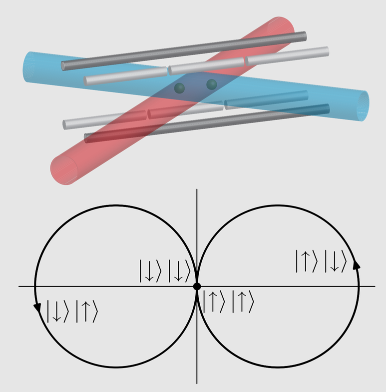
For a quantum mechanical harmonic oscillator, coherent states are the closest analogy to classical motional states. In particular, coherent states remain coherent when a force is applied: The evolution caused by a resonant force described by $H = \hbar g\hat{q}_\phi$, is $U(t) = D(-ige^{-i\phi}t)$, where $D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$ is the displacement operator of the harmonic oscillator.

For off-resonant forces, $H = \hbar g q_{\phi(t)}$, the evolution includes a geometric phase: $U(t) = \exp(i\Phi(t))D(\alpha(t))$, where $\alpha(t)$ and $\Phi(t)$ obey

$$\begin{aligned} \dot{\alpha}(t) &= -ige^{-i\phi} \\ \dot{\Phi}(t) &= \mathrm{Im}(\alpha^*(t)\,\dot{\alpha}(t)) = 2\frac{\partial}{\partial t}A(t), \end{aligned}$$

with $A(t)$ being the phase space area covered by $\alpha(t)$.

The σ_z-gate



Two ions captured in a linear Paul trap have two common axial oscillation modes: a center of mass mode, and a stretch mode where the ions move oppositely. For certain ion spacings, the two-ion Hamiltonian reads

$$H_z = \hbar\chi\hat{q}_{\phi(t)}\left(\sigma_z^{(1)} - \sigma_z^{(2)}\right),$$

corresponding to equal and opposite forces on the stretch mode if the ions are in the $|\uparrow\rangle|\downarrow\rangle$ or $|\downarrow\rangle|\uparrow\rangle$ states. Driving the stretch mode through a closed loop in phase space, conditioned on the ions being in either of these states, implements a CPHASE gate [2].